S. HAIEVSKYI, O. SHAPovalov, I. KULAKOv, O. TIMOCHKO, V. PAVlenko

MATHEMATICAL MODELS FOR CALCULATING THE RESIDUAL LIFE OF THE RECOVERABLE COMPONENTS OF THE AIRCRAFT ELECTRONIC SYSTEM

The subject matter of the article are the processes of functioning of the radio electronic system of a modern aircraft, its components, functional units and functional systems as an object for determining and calculating indicators of residual life. The goal is the analysis and improvement of the existing mathematical apparatus used to calculate the indicators of the residual resource of the aircraft radio-electronic system restored components. The tasks: To develop and generalize mathematical models for calculating the indicators of the residual life of the restored components of the aircraft radio-electronic system. Analyzed models are: models for the indicators of the residual life of the restored object of the aircraft radio-electronic system, failure flow model with a finite number of minimal updates, reliability models of the "load-strength" type. The following results were obtained: Mathematical models have been developed for calculating the indicators of residual life and residual operating time of a recoverable product with one resource element with a complete restoration of component elements of the aircraft radio-electronic system. Conclusions. Mathematical models have been developed for calculating the indicators of residual life and residual operating time of a restored product with one resource element with a complete restoration of component elements. A generalization of these models for a product restored by several resource elements during their complete restoration is obtained. Calculated ratios are obtained for the indicators of the residual resource and the residual operating time of functional devices and functional systems with a finite number of minimal updates of resource elements. Relationships are obtained for determining the limiting number of minimal restorations of functional devices and functional systems of the aircraft radio-electronic system.

Keywords: residual life; residual operating time; aircraft; mathematical model; indicator; electronic system; technical condition.

Introduction

The scheme of calculation of indicators of residual resource of radio electronic system (RES) of the plane offered in [11, 12] provides division of all accessories (AS), functional devices (FD) and functional systems (FS) of RES of the plane into non-renewable or renewable with various depth of recovery, with continuous or periodic monitoring of the technical condition, with a finite or unlimited number of failures during the specified service life, and the calculation of residual life (RL) for selected types of AS, then for FD and FS.

Literature analysis

In the scientific and technical literature, the relevant scientific and methodological support for solving the above problems is not fully developed, there are works [4, 5, 7], which consider mathematical models of complete restorations, minimal and incomplete restorations. However, their application to solve the problems of extending the resources of the aircraft fleet is almost absent.

The following are mathematical models for calculating the indicators of the residual life of non-renewable AS, renewable FD of aircraft RES for complete, incomplete and minimal restorations with a finite number of restorations and continuous monitoring of the technical condition.

In the considered mathematical models the resource is understood as "technical resource", as total operating time of a product from the beginning of its operation or its restoration after repair before transition to a limit state. The residual resource in accordance with [1] means the total operating time of the product from the moment of control of its technical condition to the transition to the limit state.

Main part

Residual resource and residual operating time of renewable products and mathematical models for calculation of their indicators at full restoration of accessories.

Under the residual life of the renewable product (FD, unit, FS or RES of the aircraft) we will understand the total operating time of the product from the moment of control of its technical condition to the resource failure. This assumes possible non-resource failures of the product, i.e. failures not related to the transition of the recoverable product to the limit state. Non-resource failures or simply product failures can be caused by the transition to non-operational state of renewable or non-renewable removable elements. Resource failure of the product is associated with the transition to the limit state of one or more resource elements. Under the resource elements of the product are those products, the expiration of which leads to the end of the product life (FD, FS or RES of the aircraft). Non-resource elements are those elements whose resource ends not earlier than the product resource, i.e. simultaneously with reaching the product limit, or after reaching the product limit, provided that this element can be used in another product of the same or similar purpose. The number of resource elements in the product may be different, and the restoration of the resource element at a particular seat may be complete, incomplete or minimal. In addition, the number of product element replacements at a particular seat cannot be unlimited. Depending on the type of elements, their load-bearing structures, other factors, it is possible to determine the maximum number of their repairs and (or) restorations. Thus, for electronic assemblies made on
printed circuit boards, the maximum number of resolders at a certain circuit position is the final value. In this case, the impossibility of restoring or repairing the FD occurs when the number of performed restorations (repairs) associated with the replacement of elements in a particular circuit position is equal to the limit \( l \), and the next failure of the element. The maximum allowable number of repairs (repairs) of the FD RES aircraft may be determined by economic constraints (\( l_{e} \)), safety requirements and (or) environmental performance (\( l_{n} \)), the actual reliability of the product (\( l_{a} \)), and other reasons. In the case of simultaneous action of several factors, the maximum allowable number of restorations of a certain circuit position can be found as the minimum of these values, i.e.

\[
l = \min \{ l_{e}, l_{n}, l_{a} \}.
\]  

The maximum allowable number of restorations \( l \) of a certain circuit position corresponds to the maximum allowable number \( p \) of its failures

\[
p = l + 1.
\]

We now formalize the RL concept for a renewable product, which includes one resource element or one circuit position, for which the maximum number of failures is set. Let \( \xi_{j} \) is the operating time of the selected circuit position of the product between failures (fig. 1);

\[
X_{j} = \sum_{j=1}^{l} \xi_{j}
\]

is the total operating time of the circuit position to the i-th failure. Then \( X_{i+1} = X_{p} \) is the total operating time of the selected circuit position to the resource failure of the product.

The residual life of the product after the moment \( \tau \) of control of its technical condition (fig. 1) is determined by the ratio:

\[
\xi_{p} = \begin{cases} 
X_{i+1} - \tau, & \tau < X_{i+1}, \\
0, & \text{if } X_{i+1} \leq \tau.
\end{cases}
\]  

Let’s find the RL distribution function of the product. Since the \((l+1)\)-th failure of the selected circuit position is a resource failure of the product, the distribution function \( F_{l+1}(t) \) is a function of the distribution of the product resource. Then we get

\[
F_{p}(t) = P(\xi_{p} < t) = \frac{F_{p}(t + \tau) - F_{p}(\tau)}{1 - F_{p}(\tau)}.
\]  

Now let’s find the probability of the event \( \xi_{p} > \tau \), i.e. the probability that the value of RL will be not less than the specified operating time \( t \) or otherwise the probability of operation of the product without resource failures during operation \( t \), starting from the moment \( \tau \), provided the product operates on the segment \([0, \tau]\) without resource failures. From (4) we have

\[
P_{p}(t) = P(\xi_{p} > t) = 1 - F_{p}(t) \quad \text{or} \quad P_{p}(t) = \frac{F_{p}(t + \tau)}{F_{p}(\tau)}.
\]

RL indicators such as average RL \( R(\tau) \) and gamma-percentage residual life \( R_{p}(\tau) \) of the recoverable product can be found by substituting in them instead of the probability of failure-free operation \( P(t) \) the probability of operation of the recoverable product without resource failures during operation \( t \), i.e. \( P_{l+1}(t) = F_{l+1}(t) \) where the distribution function \( F_{l+1}(t) \) is found by expression (3) or is a distribution function operating time before
The event $\xi_{yi}$ can happen in $l+1$ ways:

1) by the time $\tau + y$ there was no failure of the circuit position, the probability of this event is equal to $F(\tau + y)$;

2) on the interval $(u, u + du)$, there was an $i$-th failure of the circuit position, $i = 1, 2 \ldots l$ (the probability of this event is equal to $f_i(u)du$) and after this moment $\tau + y$, there were no failures until the moment $\tau + y$ (the probability of this event is equal to $\overline{F}(\tau + y - u)$).

Integrating the product $\overline{F}(\tau + y - u) \times f_i(u)du$ over all $u(0 \leq u \leq \tau)$, we obtain the probability of an event $\xi_{yi} > y$, provided that there were $i$ failures on the interval $(0, \tau)$.

Summarizing the found probabilities at $i$ for all $i = 1, 2 \ldots l$ we obtain the desired probability

$$P(\xi_{yi} > y) = \overline{F}(\tau + y) + \sum_{i=1}^{l} \int_{0}^{\tau} \overline{F}(\tau + y - u)f_i(u)du. \quad (6)$$

Formula (6) shows that the probability $P(\xi_{yi} > y)$ is a non-stationary coefficient of readiness of the recoverable product with a limited number of complete recoveries. Therefore, in the future we will denote this probability $K_z(\tau, y)$, assuming that the value $y$ is the total failure-free operation of the circuit position in the interval $(\tau, \tau + y)$, where $\tau$ is the value of the assigned resource.

We now find the function of the distribution of the residual operating time of the selected circuit position of the product to failure. Since $P(\xi_{yi} < y) = 1 - P(\xi_{yi} > y)$, from formula (6) follows

$$P(\xi_{yi} < y) = \frac{1}{l} \sum_{i=1}^{l} \int_{0}^{\tau} \overline{F}(\tau + y - u)f_i(u)du. \quad (7)$$

Other indicators of the reliability of the selected circuit position in the RO interval, used to solve problems of continuation of resources, is the average RO and gamma-percent RO $T_{\gamma}(\tau)$. For the selected circuit position, we will find these indicators by the ratio

$$T(\tau) = \int_{0}^{\infty} P(\xi_{yi} > y)dy, \quad (8)$$

$$P(\xi_{yi} > T_{\gamma}(\tau)) = 0.01\gamma \quad (9)$$

Let’s consider the asymptotic behavior of the RO of the selected circuit position, which determines the resource of the recoverable product, i.e. we investigate the behavior of a random variable $\xi_{yi}$ at $\tau \to \infty$ and $l \to \infty$.

Since $l \to \infty$ then

$$\lim_{\tau \to \infty} P(\xi_{yi} > y) = \lim_{\tau \to \infty} \int_{0}^{\tau} \overline{F}(\tau + y - u)\omega(u)du$$

By the Smith’s theorem [3] we find that this limit is equal to

$$P(\xi_{yi} > y) = \frac{1}{T_0} \int_{0}^{\infty} \overline{F}(\tau + y)\omega(\tau)d\tau = \frac{1}{T_0} \int_{0}^{\infty} \overline{F}(y)dz. \quad (10)$$

As a result, we obtain the distribution of stationary residual operating time of the selected circuit position of the restored product. In particular, for $\tau \to \infty$ and $l \to \infty$ the value of the average RO $T(\infty)$ is found by the formula.
The residual operating time of the recoverable product:

\[ T(\infty) = \int_0^{\infty} P(\xi_H > y) \, dy = \frac{T_0}{2} + \frac{\sigma^2}{2T_0}. \]  

(11)

Note that expressions (10) and (11), and not expressions (6), (8), (9) are used in engineering practice. However, in the tasks of continuing the RES resources of the aircraft, we are more interested in the non-stationary interval of operation of AS, FD, FS RES of aircraft and its components, i.e. \( t \leq (0,1-0,3)T_0 \), where \( T_0 \) is the average failure time of the device, unit, AS or AE. Therefore, the calculations of RL and RO products must be performed for the non-stationary interval of their operation according to the formulas for non-stationary RL indicators and other reliability indicators in the RO interval.

Consider how to calculate the indicators of the recovery of a renewable product with one circuit position, which determines the resource of the product.

Let’s denote \( Y_j \) as the operating time of the renewable product to the \( j \)-th failure, \( f_{jj}(t) \) is the density of the distribution of the random variable \( Y_j \), \( X_p \) is the operating time of the selected circuit position to the resource failure. Then the RO of the product to the next failure (fig. 3) can be determined by the ratio:

\[ e^{\prod}_{\tau_j} = \frac{Y_j - \tau \text{,} \, \{ Y_{j-1} - \tau \} \cap Y_j > \tau \cap Y_j \leq X_p \}, \, j \leq 1, \prod; \, 0, \text{in other cases}. \]

Residual operating time of the recoverable product before failure differs from the definition (2) by the presence of an additional condition: the event \( \{ Y_j < X_p \} \) that the accidental operating time of the product before the next \( j \)-th failure should not exceed the total operating time of the product (circuit position determining the product resource) to resource failure. Therefore, the above indicators of the residual resource (6), (7), (9) of the type “probability”, in terms of the situation under consideration, are conditional. To calculate the unconditional indicators of the recovery of the recoverable product to the next \( j \)-th failure, we find the probability of the event (conditions) \( P\{Y_j < X_p\} \). We have

\[ P\{Y_j < X_p\} = \int_0^{\infty} F_{p}(t) f_{jj}(t) \, dt, \]

where \( f_{jj}(t) \) is the density of the distribution of the operating time of the product to the \( j \)-th failure; \( F_{p}(t) \) is the function of the distribution of the operating time of the circuit position, which determines the resource of the product, to the resource failure.

To calculate this probability, we can use mathematical models of reliability of the type “load-strength” for different functions of load distribution and strength [9]. Then the probability that the RO of the recoverable product will be not less than the set \( y \), we find by the ratio

\[ P\left(\xi > y\right) = F\left(\tau + y\right) + \frac{\Gamma}{\sum_{i=0}^{\infty} P\{Y_i < X_p\} f_{j}(\tau + y - u) f_j(u) \, du}. \]

(12)

Other indicators of RO, for example average RO, gamma-percent RO of the restored product is calculated by a ratio (8), (9) by substitution in them of the unconditional probability \( P\{Y_j < X_p\} \) \( \xi > y \) found by the formula (12).

We obtain calculation formulas for determining the indicators of RO for the case when the operating time between failures of the selected circuit position, which determines the resource failure of the product, has a truncated normal distribution with mathematical expectation \( T_0 \) and standard deviation \( \sigma \), i.e.

\[ \xi_{4} = 0 \quad X_{11} \quad X_{12} \quad X_{13} \quad X_{14} \quad \text{a)} \]

\[ \xi_{4} = 0 \quad X_{21} \quad X_{22} \quad X_{23} \quad X_{24} \quad \text{b)} \]

\[ \xi_{4} = 0 \quad X_{31} \quad \text{c)} \]

\[ \xi_{4} = 0 \quad Y_1 \quad Y_2 \quad Y_3 \quad Y_4 \quad \text{d)} \]

**Fig. 3.** Determination of residual operating time of the recoverable product: a) the failure flow of the 1st circuit position, which determines the resource of the product; b), c) failure flows of the 2nd and 3rd circuit positions, which do not determine the resource of the product; d) product failure flow; \( X_{ij} \) - development of \( i \)-th schematic position to \( j \)-th failure; \( e^{\prod}_{\tau_6} \) - residual operating time to the next (sixth) failure; \( \xi_{4} \) - residual operating time of the circuit position, which determines the resource of the product, to the resource failure.
F(\xi_t > y) = \sum_{i=1}^{n} a_i \Phi\left(\frac{T_0 - \tau - y}{\sigma_i}\right) + \int_{-\infty}^{\infty} \Phi\left(\frac{u-T_i}{\sigma_i}\right) f_i(u) du.

(13)

To calculate the probability

P(T_i > y) = a_i \Phi\left(\frac{T_0 - \tau - y}{\sigma_i}\right) + \int_{-\infty}^{\infty} \Phi\left(\frac{u-T_i}{\sigma_i}\right) f_i(u) du.

(14)

We reduce the calculation of the integral in expression (13) to the basic Owen integrals. From expressions (13) and (14) we have

x = \frac{u-T_i}{\sigma_i}, a + bx = \frac{T_0 - \tau - y - u}{\sigma}, \quad a = \frac{T_0 - T_u - \tau - y}{\sigma_i}, \quad b = \sigma_i.

\int_{-\infty}^{\infty} \Phi\left(\frac{u-T_i}{\sigma_i}\right) f_i(u) du = \int_{0}^{\infty} \Phi\left(\frac{1}{2} \phi(x)\right) dx.

Let us now use the properties of a definite integral and the property of an Owen integral: T(h, a) = -T(h, a).

Let’s consider two cases: \tau < T_{0i} and \tau > T_{0i}. Get for \tau < T_{0i}:

\int_{-T_i/\sigma_i}^{(T_i-\tau)/\sigma_i} \Phi(a + bx) \phi(x) dx = \int_{0}^{(T_i-\tau)/\sigma_i} \Phi(a + bx) \phi(x) dx - \int_{0}^{(T_i-t)/\sigma_i} \Phi(a + bx) \phi(x) dx.

(16)

and for \tau > T_{0i}:

\int_{-T_i/\sigma_i}^{(T_i-\tau)/\sigma_i} \Phi(a + bx) \phi(x) dx = \int_{0}^{(T_i-\tau)/\sigma_i} \Phi(a + bx) \phi(x) dx + \int_{0}^{(T_i-t)/\sigma_i} \Phi(a + bx) \phi(x) dx.

(17)

The results of probability calculations P(\xi_t > y) for the ratio (13) (17) show that the nature of the obtained dependences of the probability P(\xi_t > y) on the value of the residual operating time at different values of the assigned resource \tau corresponds to the expected, which indicates the correctness of the obtained ratios.

Now consider how to calculate other indicators of reliability of the restored FD (FS) of the aircraft RES. According to the known structural scheme of reliability S(x) of the FC of the aircraft RES, the reliability indicators are calculated on the interval of residual operating time, by substituting into the function of the previously calculated reliability indicators of the FD (FS) elements of the aircraft RES:

R_i(\tau, y) = S\left(K_{i1}(\tau, y), K_{i2}(\tau, y), ..., K_{ip}(\tau, y), ..., P_j(\tau + y), ...ight).

(18)

where K_{i\tau}(\tau, y) is a non-stationary coefficient of readiness of the i-th renewable FC element FU; P_j(\tau + y) is the probability of failure-free operation (\tau + y) for the operation of uncontrolled and non-renewable j-th element in the considered time interval.

In this case, the controlled elements, in turn, are divided into continuously controlled and periodically controlled, renewable with varying degrees of resource recovery: FR, IR and MR. Next is a block diagram of the reliability of the FD (FS) of the aircraft RES.
If necessary, calculate other indicators, for example, the average residual operating time \( FD \) (\( FS \)) in relation (8), gamma-percentage residual operating time \( FD \) (\( FS \)) according to formula (9).

Note that the developed approach is focused on the fact that during the next current repair of \( FD \) (\( FS \)) of the aircraft \( RES \) restores the resource (in full at \( FR \), partially - at \( IR \) and at zero at \( MR \)) \( AE \) \( FD \) (\( FS \)) only in the variable (or renewable) \( AE \). The resource of other \( AE \) \( FD \) (\( FS \)) is not restored. The difference between other approaches to calculating the reliability of \( RES \) that is the recovery of \( PC \) \( FD \) (\( FS \)) by replacing failed \( AE \) involves complete recovery of the resource of all \( FD \) (\( FS \)), which is unacceptable in the case of calculating the indicators of \( OR \) and \( ON \) when solving resource renewal tasks.

To perform calculations on the ratio (18) it is necessary to pre-calculate the coefficients of operational readiness of the elements of \( FD \) (\( FS \)), the resource of which is restored in full, incomplete or minimal. We use for this purpose the calculated relations for the failure flow parameter obtained in [11] for flows with \( FR \), \( IR \) and \( MR \) with a limited number of recoveries. Thus, for the case of \( FR \), the probability that the residual operating time before the failure of the element of a certain circuit position, \( FD \), \( FS \) will be not less than a given value, we find the formula

\[
K_{s}^{(1)}(\tau, y) = \frac{1}{u} P(\tau + y) + \int_{0}^{\tau} P(\tau + y - u) \alpha_{h}(u) du \, , \quad (19)
\]

or substituting in (19) we obtain

\[
K_{s}^{(1)}(\tau, y) = P(\tau + y) + \sum_{k=0}^{l_{k}} P(\tau + y - u) f_{k}(u) du \, , \quad (20)
\]

where \( l_{k} \) is the maximum possible number of complete \( AE \) recoveries for the considered total operating time \( (\tau + y) \).

We now write the corresponding formula for the case of the \( MR \) element. In [11] the basic relations for processes with instantaneous \( MR \) were considered. Then, conducting reasoning similar to the above in deriving formula (6), we obtain

\[
K_{s}^{(2)}(\tau, y) = P(\tau + y) + \sum_{k=1}^{l_{k}} P(\tau + y - u) f_{k}^{(2)}(u) du \, , \quad (21)
\]

where \( P(\tau + y - u) \) is the probability of failure of the minimally recoverable element \( FD \) (\( FS \)) in the interval \( (\tau + y - u) \); \( f_{k}^{(2)}(u) \) is the distribution density of the operating time of the element to the \( k \)-th \( MR \); \( l_{2} \) is the maximum possible number of \( MR \) element \( FD \) (\( FS \)) for the considered operating time.

Substituting in (21) the appropriate formulas for the process with instant recovery, we obtain the following relationship

\[
K_{s}^{(2)}(\tau, y) = e^{-\lambda(\tau + y)} + \sum_{k=1}^{l_{k}} \frac{\lambda(\tau + y)}{k!} e^{-\lambda(\tau + y)} f_{k}^{(2)}(u) du \, , \quad (22)
\]

where \( \lambda(\tau + y) = \int_0^\tau \lambda(x) dx \).

After performing the transformation, we obtain the final expression for the probability that the residual operating time of the minimum renewable element of a certain circuit position \( FD \) (\( FS \)) RES of the aircraft will not be less than the specified value \( y \).

For flows with \( IR \) we have

\[
K_{s}^{(3)}(\tau, y) = P(\tau + y) + \sum_{k=1}^{l_{k}} P_{k+1}(\tau + y - u) f_{k}^{(3)}(u) du \, , \quad (23)
\]

where \( P_{k+1}(\tau) \) is the probability of failure of the element during operation \( x \) after the \( k \)-th incomplete recovery; \( f_{k}^{(3)}(u) \) – operating time distribution density to the \( k \)-th \( IR \).

These calculated ratios allow to calculate the residual life and other indicators of the reliability of the renewable \( FD \) (\( FS \)) for an extended period of operation for the case when there is one circuit position that determines the resource \( FD \) (\( FS \)).

Consider now the case where the number of circuit positions that determine the life of the renewable \( FD \) or \( FS \) RES aircraft, more than one. Further we will not make distinctions between a resource element of a product and the schematic position defining a product resource.

Let \( M \) is the set of resource elements \( FD \) (\( FS \)).

Let's divide this set into disparate subsets \( M_{s}, s = 1, n \), based on the same number of maximum allowable number of replacements \( l_{s} \) and the same characteristics of failure (the second condition is not required). For the operation of "splitting" the set of resource elements \( M \), the relations are performed

\[
M = M_{1} \cup M_{2} \cup \ldots \cup M_{n} \, ,
\]

\[
M = M_{1} \cap M_{2} \cap \ldots \cap M_{n} = \emptyset \, .
\]

Let \( R_{ys} \) is the lower estimate of the value of the gamma-percent resource of the elements of the subset \( M_{y} \). Then the lower estimate of the gamma percentage resource \( FD \) can be found by the formula

\[
R_{y} = \min_{s} \{ R_{y,s} \} \, . \quad (24)
\]
A similar relationship can be written for the lower estimate of the gamma percentage residual FD resource

\[ R_\gamma (\tau) = \min \left\{ R_{\gamma s} (\tau) \right\}. \]  

It is assumed that the resource failure of the element limits the reliability of any subset \( M_s \) leads to the resource failure of the FD (FS). Then the value \( R_\gamma \) (or \( R_\gamma (\tau) \)) is determined by the time during which there will be no resource failure of the elements of the subset \( M_s \), the moment of which, in turn, is determined by the earliest moment of occurrence of the \((l_i + 1)\)-th element failure for all elements that make up the subset \( M_s \).

Let the subset \( M_s \) consist of \( \left| M_s \right| = m_s \) elements that limit the reliability of the FD (FS) RES of the aircraft. We will consider the distribution functions as a set \( m_s \) of independent random variables \( \{T_1, T_2, \ldots, T_{m_s}\} \) that represent the development of circuit positions of a subset \( M_s \) of elements to resource failures. Let’s find the distribution function \( G_s (t) \) and the distribution density \( g_s (t) \) of a random variable \( T(s) \):

\[ T(s) = \min \left\{ T_1, T_2, \ldots, T_{m_s} \right\}, \]  

which is a random run of a subset of FD (FS) elements to a resource failure. We have:

\[ G_s (t) = \sum_{i=1}^{m_s} P(T_i > t) = \sum_{i=1}^{m_s} \prod_{j=1}^{m_s} (1 - F(t_i)), \]

\[ g_s (t) = \sum_{j=1}^{m_s} f_j(t) \prod_{i=1}^{m_s} \left[ 1 - F(t_i) \right] / \prod_{i=1}^{m_s} \left[ 1 - F(t) \right]. \]  

Random value \( T_i \) in relation (26) is a random operation of the \( i \)-th circuit position

\[ T_i = \sum_{k=1}^{l_i+1} t_{ki}, \]

where \( t_{ki} \) is the operating time of the element at a certain \( i \)-th circuit position to \( k \)-th replacement.

In the case of complete restorations, random variables \( t_{ij} \) can be considered as independent equally distributed random variables, and the random variable distribution function \( T_i \) can be found as a convolution of the \((l_i + 1)\)-th order of random variables. If all elements of the subset \( M_s \) have the same functions of distribution \( F_i (t) \) of a random variable \( T_i \), then it follows from expression (27)

\[ G_s (t) = 1 - \left( 1 - F_i (t) \right)^{m_s} = 1 - \overline{F_i} (t)^{m_s}, \]

\[ g_s (t) = m_s f_s (t) \overline{F_s} (t)^{m_s-1}. \]  

The relation for the residual resource distribution function of the subset \( M_s \) of FD elements is obtained by conducting similar considerations for the residual resource distribution function \( F_{i_{\infty}} (t) \) of the element at the \( i \)-th circuit position

\[ F_{i_{\infty}} (t) = \frac{F_p (t + \tau) - F_p (\tau)}{1 - F_p (\tau)} \]

and the probability that during the residual operating time \( t \) there will be no failures of the \( i \)-th circuit position element

\[ F_{i_{\infty}} (t) = 1 - F_{i_{\infty}} (t). \]

Then the distribution function of the residual resource of the subset of elements \( M_s \) is found by the relation (28): 

\[ G_s (t) = 1 - \left( P_{\gamma s} (t) \right)^{m_s}. \]  

We now obtain the calculated ratios for the quantities \( R_{\gamma s} \) and \( R_{\gamma s} (\tau) \).

From relation (28) and the definition of the gamma-percent resource follows \( P(T(s) > Y_{\gamma s}) = 1 - G_s (Y_{\gamma s}) = \gamma \) or

\[ \overline{F_i} (Y_{\gamma s}) = \gamma^{m_s}. \]  

Solving equation (30) according \( R_{\gamma s} \) to the given value \( \gamma \) and various parameters \( m_s \) of all subsets \( M_s \) of FD (FS) elements, we obtain the corresponding values of gamma-percent resources. Next at the expression (24) we find the lower estimate of the gamma-percent resource FD (FS) of the aircraft RES.

The calculated ratios for the FD gamma-percent residual resource are obtained by conducting similar considerations for the distribution function of the OP \( F_{i_{\infty}} (t) \) of the \( i \)-th circuit position. From expression (29) it follows that

\[ P_{\infty} \left( R_{\gamma s} (\tau) \right) = \gamma^{m_s}, \]  

or

\[ P_{\infty} \left( R_{\gamma s} (\tau) + \tau \right) = P_{\infty} (\tau) \gamma^{m_s}, \]  

where \( P_{\infty} (\tau) = 1 - F_{\gamma s} (\tau) \).
Solving equation (32) according to \( R_{ys} \) for different subsets \( M_s \) of FD (FS) of the aircraft RES, we obtain the corresponding values \( R_{ys}(\tau) \). The lower estimate of the gamma-percentage residual resource FD can be found from the expression (25).

We now obtain the calculated ratios for the gamma-percentage residual resource of subsets \( M_s \) of FD elements on the example of different functions of the distribution of operating time to failures of its elements.

A. The operating time before the failure of the elements of the subset \( M_s \) has a normal distribution with a mathematical expectation \( \mu_s \) and standard deviation \( \sigma_s \), and \( \mu_s > 3\sigma_s \). Then the function of the distribution of the operating time of these elements to the resource failure has the form

\[
F_s(t) = \Phi \left( \frac{t - \mu_s (l_s + 1)}{\sigma_s \sqrt{l_s + 1}} \right),
\]

where \( \Phi(z) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{z} e^{-\frac{y^2}{2}} dy \).

Equation (30) for this distribution function is as follows:

\[
\Phi \left( \frac{\mu_s (l_s + 1) - R_{ys}}{\sigma_s \sqrt{l_s + 1}} \right) = e^{\gamma m_s}.
\]

(33)

Let’s mark \( \alpha_s = \gamma m_s \). Solving equation (33) as for the value \( R_{ys} \) we obtain

\[
R_{ys} = \mu_s (l_s + 1) - U_{\alpha_s} \sigma_s \sqrt{l_s + 1},
\]

(34)

where \( U_{\alpha_s} \) is an \( \alpha_s \)-quantile of normal distribution.

Equation (32) for finding the gamma-percentage residual resource has the form:

\[
\Phi \left( \frac{\mu_s (l_s + 1) - R_{ys} - \tau}{\sigma_s \sqrt{l_s + 1}} \right) = \gamma m_s. \Phi \left( \frac{\mu_s (l_s + 1) - \tau}{\sigma_s \sqrt{l_s + 1}} \right).
\]

(35)

Let’s mark \( \alpha_s(\tau) = \gamma m_s. \Phi \left( \frac{\mu_s (l_s + 1) - \tau}{\sigma_s \sqrt{l_s + 1}} \right) \).

The solution of equation (35) has the form

\[
R_{ys}(\tau) = \mu_s (l_s + 1) - \tau - U_{\alpha_s(\tau)} \sigma_s \sqrt{l_s + 1}.
\]

(36)

B. The operating time before failure of the elements of the subset \( M_s \) is distributed exponentially with the parameter \( \lambda_s \). Consideration of such distribution for an estimation of indicators of a residual resource represents more theoretical interest, the results of calculations received thus can be used for comparison with results of calculations on VFI-distributions.

The function \( F_s(\tau) \) for subset \( M_s \) elements has the form

\[
F_s(\tau) = \frac{l_s!}{k!} \exp(-\lambda_s \tau).
\]

(37)

Substituting (37) into (30) we obtain

\[
\sum_{k=0}^{\infty} \frac{\lambda_s^{R_{ys}}} {k!} e^{-\lambda_s R_{ys}} = \frac{1}{m_s}.
\]

(38)

To solve equation (38) we use the Poisson distribution \( \Theta_k = \frac{\alpha^k e^{-\alpha}}{k!} \), the tables we have. To do this, multiply both parts of equation (38) by (-1) and add 1. As a result, we obtain

\[
\sum_{k=0}^{\infty} \frac{\alpha^k e^{-\alpha}}{k!} = 1 - \gamma m_s.
\]

(39)

where

\[
\alpha = \lambda_s R_{ys}.
\]

(40)

Then, for known quantities \( \Theta_k, k=1, 2, 3, \ldots, l_s + 1 \) we take \( \frac{1}{m_s} \), and the desired value \( R_{ys} \) is found from the expression (40): \( R_{ys} = \frac{\alpha}{\lambda_s} \).

Equation (32) for finding the gamma-percentage of OR has the form

\[
\sum_{k=0}^{l_s} \frac{\lambda_s^{R_{ys}}}{k!} \exp(-\lambda_s [R_{ys}(\tau) + \tau]) = \gamma m_s \sum_{k=0}^{l_s} \frac{\lambda_s^{R_{ys}}}{k!} \exp(-\lambda_s \tau) + \gamma m_s \sum_{k=0}^{l_s} \frac{\lambda_s^{R_{ys}}}{k!} \exp(-\lambda_s \tau).
\]

(41)

Let’s mark \( \beta_s(\tau) = \gamma m_s \sum_{k=0}^{l_s} \frac{\lambda_s^{R_{ys}}}{k!} \exp(-\lambda_s \tau) \).

Next, performing transformations similar to the above, we obtain the equation

\[
\sum_{k=l_s+1}^{\infty} \frac{\alpha}{k!} e^{-\alpha} = 1 - \beta_s(\tau),
\]

(42)

where

\[
\alpha = \lambda_s [R_{ys}(\tau) + \tau].
\]

(43)
Then for the values $1 - \beta_k(\tau)$ and $I_k + 1$ according to Poisson tables we find the parameter $\alpha$. The desired value of gamma-percentage RL is found as $R_{\gamma x} = \frac{\alpha}{\lambda_x} - \tau$.

Other reliability indicators of FD (FS) of the aircraft RES in the interval of residual operating time are found according to the block diagram given in [12].

Conclusions

1. Mathematical models for calculation of indicators of residual resource and residual operating time of the restored product with one resource element at full restoration of accessories are developed. The generalization of these models for the product which is restored by several resource elements at their full restoration is received.

2. The calculated ratios for the indicators of the residual resource and the residual operating time of functional devices and functional systems with a finite number of minimal renewals of resource elements are obtained. A relation is obtained to determine the limit number of minimum restorations of functional devices and functional systems of the aircraft electronic system.

References


Received 27.08.2020

Відомості про авторів / Сведения об авторах / About the Authors

Гасьєвський Сергій В'ячеславович – Кіровоградська льотна академія Національного авіаційного університету, аспірант кафедри льотної експлуатації, аеродинаміки та динаміки польоту, Кропивницький, Україна; email: snegovik2207@ukr.net; ORCID: https://orcid.org/0000-0003-3434-7494.

Гасьєвський Віктор Володимирович – Кіровоградська льотна академія Національного авіаційного університету, аспірант кафедри льотної експлуатації, аеродинаміки та динаміки польота, Кропивницький, Україна.
Куляков Ігор Павлович – Командування Сил Логістики Збройних Сил України, головний спеціаліст, Київ, Україна; email: igor.kulakov@ukr.net; ORCID: https://orcid.org/0000-0002-7392-8876.

Куляков Ігор Павлович – Командування Сил Логістики Вооружених Сил України, головний спеціаліст, Київ, Україна.

Kulakov Igor – Logistic Command of Armed Forces of Ukraine, Senior Specialist, Kyiv, Ukraine.

Шаповалов Олександр Васильович – кандидат технічних наук, Харківський національний університет Повітряних Сил ім. І. Кожедуба, старший викладач кафедри математичного та програмного забезпечення АСУ, Харків, Україна; email: kpakokot@gmail.com; ORCID: https://orcid.org/0000-0002-9744-9431.

Шаповалов Олександр Васильович – кандидат технічних наук, Харківський національний університет Воздушних Сил ім. І. Кожедуба, старший преподаватель кафедры математического и программного обеспечения АСУ, Харьков, Украина.

Shapovalov Oleksandr – PhD (Engineering Sciences), Ivan Kozhedub National Air Force University, Senior Lecturer of the Department of Mathematical and Software ACS, Kharkiv, Ukraine.

Тімочко Олександр Володимирович – кандидат технічних наук, компанія "Kreditech", старший розробник, Гамбург, Німеччина; email: alexander.timochko@gmail.com; ORCID: https://orcid.org/0000-0003-0424-0426.

Тімочко Олександр Володимирович – кандидат технічних наук, компанія "Kreditech", старший разработчик, Гамбург, Германия.

Timochko Oleksandr – PhD (Engineering Sciences), Company "Kreditech", Senior Developer, Hamburg, Germany.

Павленко Владислава Максимівна – Харківський національний університет ім. В. М. Каразина, студентка кафедри прикладної математики, Харків, Україна; email: marnidor@gmail.com; ORCID: https://orcid.org/0000-0003-0976-0252.

Павленко Владислава Максимівна – Харківський національний університет ім. В. Н. Каразина, студентка кафедри прикладної математики, Харьков, Украина.

Pavlenko Vladislava – V. N. Karazin Kharkiv National University, Student of the Department of Applied Mathematics, Kharkiv, Ukraine.

**МАТЕМАТИЧНІ МОДЕЛІ ДЛЯ РОЗРАХУНКУ ПОКАЗНИКІВ ВІДНОВЛЮВАНИХ ВИРОБІВ РАДІОЕЛЕКТРОННОЇ СИСТЕМИ ЛІТАКА**

Предметом вивчення в статті є процеси функціонування радіоелектронної системи сучасного літака, її комплектуючих елементів, функціональних вузлів та функціональних систем як об’єкта визначення та розрахунку показників залишкового ресурсу. Метою є аналіз та вдосконалення існуючого математичного апарату, що застосовується для розрахунку показників залишкового ресурсу відновлювальних комплектуючих виробів радіоелектронної системи літака. Завдання: Розробити та узагальнити математичні моделі для розрахунку показників залишкового ресурсу відновлювальних комплектуючих виробів радіоелектронної системи літака. Аналізовані моделі є: моделі для показників залишкового ресурсу відновлюваного об’єкта радіоелектронної системи літака, модель потоку відмов з кінцевим числом мінімальних відновлень, моделі надійності типу "нагрузка – відповідь". Отримані такі результати: Розроблено математичні моделі для розрахунку показників залишкового ресурсу відновлюваного виробу з одним ресурсним елементом при повному відновленні комплектуючих елементів радіоелектронної системи літака. Висновки: Розроблено математичні моделі для розрахунку показників залишкового ресурсу відновлюваного виробу з одним ресурсним елементом при повному відновленні комплектуючих елементів радіоелектронної системи літака. Складається на виправлення гранічного числа мінімальних відновлень функціональних пристроїв і функціональних систем радіоелектронної системи літака.

**МАТЕМАТИЧЕСКИЕ МОДЕЛИ ДЛЯ РАСЧЕТА ПОКАЗАТЕЛЕЙ ОСТАТОЧНОГО РЕСУРСА ВОССТАНАВЛИВАЕМЫХ КОМПЛЕКТУЮЩИХ ИЗДЕЛИЙ РАДИОЭЛЕКТРОННОЙ СИСТЕМЫ САМОЛЕТА**

Предметом изучения в статье являются процессы функционирования радиоэлектронной системы современного самолета, ее комплектующих элементов, функциональных узлов и функциональных систем как объекта определения и расчета показателей остаточного ресурса. Целью является анализ и совершенствование существующего математического аппарата, применяемого для расчета показателей остаточного ресурса восстанавливаемых комплектующих изделий радиоэлектронной системы самолета. Задачи: Разработать и обобщить математические модели для расчета показателей остаточного ресурса восстанавливаемых комплектующих изделий радиоэлектронной системы самолета. Анализируемыми моделями являются: модели для показателей остаточного ресурса восстанавливаемого объекта радиоэлектронной системы самолета, модель потока отказов с конечным числом минимальных обновлений, модели надежности типа "нагрузка – прочность". Получены следующие результаты: Разработаны математические модели для расчета показателей остаточного ресурса и остаточной наработки восстанавливаемого изделия с одним ресурсным элементом при полном восстановлении комплектующих элементов радиоэлектронной системы самолета. Выводы: Разработаны математические модели для расчета показателей остаточного ресурса и остаточной наработки восстанавливаемого изделия с одним ресурсным элементом при полном
восстановлении комплектующих элементов. Получено обобщение этих моделей для изделия, восстанавливаемого несколькими ресурсными элементами при их полном восстановлении. Получены расчетные соотношения для показателей остаточного ресурса и остаточной наработки функциональных устройств и функциональных систем при конечном числе минимальных обновлений ресурсных элементов. Получены соотношения для определения предельного числа минимальных восстановлений функциональных устройств и функциональных систем радиоэлектронной системы самолета.

**Ключевые слова:** остаточный ресурс; остаточная наработка; самолет; математическая модель; показатель; радиоэлектронная система; техническое состояние.

**Библиографические описи / Bibliographic descriptions**
