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STACKING HYBRID SYSTEM OF COMPUTATIONAL INTELLIGENCE BASED ON KERNEL ACTIVATION-MEMBERSHIP FUNCTIONS AND ITS ONLINE LEARNING IN PATTERN RECOGNITION TASK

Subject of research. The subject of the research is a stacking hybrid computational intelligence system based on a kernel activation-membership function, which combines the advantages of neural networks and fuzzy models for solving pattern recognition tasks under conditions of uncertainty, noise, and limited dimensionality of data sets. **The goal of the work.** The goal of the work is to develop a stacking hybrid system with a kernel activation-membership function and online learning to improve the accuracy, stability, and speed of pattern recognition in non-stationary environments in real-time. **Task.** Develop the architecture of a stacking hybrid system; propose a kernel activation-membership function and an algorithm for its parameterization; construct an online learning algorithm with step-by-step parameter updates; investigate the properties of convergence and computational complexity; conduct an experimental comparison with basic machine learning models and classical neuro-fuzzy approaches. **Methods.** Machine learning methods, ensemble and stacking approaches, kernel methods, adaptive optimization in online mode, statistical analysis of classification quality, modelling and computational experiments on synthetic and real datasets applied to pattern recognition tasks. **Results.** The proposed model provides higher classification accuracy and better generalization ability compared to individual baseline models, demonstrates resistance to noise and changes in data distributions, as well as rapid adaptation in streaming data conditions thanks to online learning. It is shown to reduce the error and ensure stable operation of the system in non-stationary environments. **Conclusions.** The developed stacking hybrid system with a kernel activation-membership function is an effective tool for real-time pattern recognition tasks. The combination of the stacking approach and online learning increases the accuracy, stability and adaptability of the system, which makes the proposed approach promising for practical application in intelligent information systems and cyber-physical environments.

Keywords: computational intelligence; data stream mining; kernel activation-membership function; machine learning; neuro-neo-fuzzy system; stacking hybrid system.

Introduction

Computational intelligence (CI) systems have become popular today for solving various information processing problems [1–3] and, above all, classification problems – pattern recognition, mainly images, which is explained from a formal point of view by their universal approximating capabilities. Here, the most widespread are deep convolutional neural networks (DCNN) [4–6], which allow restoring nonlinear separating multidimensional hypersurfaces of any complex shape between an arbitrary number of possible classes. However, some problems may arise in the case of overlapping classes when observations can simultaneously belong to several classes with different levels of membership. In this case, hybrid systems of computational intelligence (HSCI) and, above all, neuro-fuzzy systems are the most effective.

Analysis of modern publications

At the same time, deep systems are quite cumbersome with a fixed multilayer architecture, containing a very large number of tuned parameters and requiring too large volumes of training samples for their learning, which are not always available when solving many practical tasks. In addition, the stationarity of the characteristics of the objects forming this sample is assumed a priori. One way or another, traditional DCNNs are quite complex and slow systems.

Essentially faster in terms of the learning process are systems (primarily neural networks) with kernel activation functions [7], the most common of which are Radial Basis Function Networks (RBFNs) [8–11] which are also characterized by universal approximating properties, but are able to adjust their synaptic weights online. This is explained by the fact that the output signal of these networks linearly depends on the adjusted

parameters that allows the use of speed-optimized learning algorithms, including the Gaussian Newtonian ones.

In [12], a convolutional RBFN was proposed, which demonstrated high speed and quality of learning compared to DCNNs based on elementary Rosenblatt perceptrons.

At the same time, traditional RBFNs suffer from so-called "curse of dimensionality" effect when the number of kernel activation functions (R -neurons) grows rapidly with the increase in the dimensionality of the input signal-image. Finding the required number of R -neurons in the network could be done using the methods of evolutionary computational intelligence systems [13–15], however the gradual addition of kernel functions to the network [16] slows down the learning process.

In our opinion, some of the noted problems can be overcome by using the stacking approach [17], when the system as a whole is formed from separate subsystems, each of which is either a neural network or a neuro-fuzzy or neo-fuzzy system [18–19] trained independently of the other subsystems using different algorithms. This approach will increase the speed of training and does not require too large volumes of training samples.

The goal of the work. The goal of the work is to improve the efficiency of a stacking hybrid system with a kernel activation-membership function and online learning to improve the accuracy, stability, and speed of pattern recognition in non-stationary environments in real-time.

Methods and Materials

Architecture of a stacking hybrid computational intelligence system based on kernel activation-membership functions

The feedforward architecture of the proposed SHSCI is shown in Fig. 1 and contains two stacks and an output layer formed by a softmax activation function. The zero receptive layer receives a vector input signal-image $x(k) = (x_1(k), \dots, x_i(k), \dots, x_n(k)) \in R^n$, where $k = 1, 2, \dots, N, N+1, \dots$ either the number of a specific observation in the training sample, or the index of the discrete current time in the case when the data for processing are received online.

In this case, it is assumed a priori that all data belong to (or are pre-coded into) some limited interval $x_{i \min} \leq x_i(k) \leq x_{i \max}, \forall i = 1, 2, \dots, n$ first stack is

essentially a simplified RBFN and is formed by $h > n$ so-called R -neurons, i.e. multidimensional kernel activation functions $h > n$. In this layer the dimensionality of the input space R^n increases which is formed in space R^h by the R -neuron system. This approach, on which all radial-basis neural networks are based according to the T.M. Cover theorem [20–21], allows transforming the initial linearly inseparable problem of pattern recognition in space R^n into linearly-separable one in a space of increased dimension R^h .

Multidimensional Gaussians are usually used as activation functions in RBFNs due, first of all, to the simplicity of their derivatives, which, in turn, simplifies the learning process

$$\varphi_l(x(k)) = u_l(k) = \varphi_l(\|x(k) - c_l\|_2^2, \sigma_l^2) = e^{-\frac{\|x(k) - c_l\|_2^2}{2\sigma_l^2}}.$$

Here the vector c_l specifies the coordinates of the center of the kernel function φ_l , and a scalar parameter σ^2 is its "width". Although theoretically any kernel (most often bell-shaped) function can be used as an activation function, it is easy to construct an RBFN with matrix inputs using Gaussians [22, 23]. This means that the input of such a network can be supplied with not only a vector signal but also a matrix signal, i.e. an image without its prior transformation using the convolution operation. In this case, the radial basis activation function can be written in the form

$$\varphi_l(x(k)) = u_l(k) = e^{\frac{(Sp(x(k) - c_l)(x(k) - c_l))^T}{2\sigma^2}},$$

where $Sp(\cdot)$ is the matrix trace symbol, is the square of the Frobenius matrix norm, which is consistent with the Euclidean vector norm and specifies the distance between the matrix input signal $x(k)$ and the matrix center of the Gaussian c_l

$$(Sp(x(k) - c_l)(x(k) - c_l))^T.$$

At the output of this stack, a vector signal of increased dimension is formed which is fed to the second stack. This vector signal is fed to a generalized neo-fuzzy neuron (GNFN) [24], which, in turn, is a multidimensional modification of the standard neo-fuzzy neuron [18, 19, 25].

$$\varphi(k) = u(k) = \varphi_1(x(k)) \dots, \varphi_l(x(k)) \dots, \varphi_h(x(k)) \dots = (u_1(k) \dots, u_l(k) \dots, u_h(k))^T \in R^h.$$

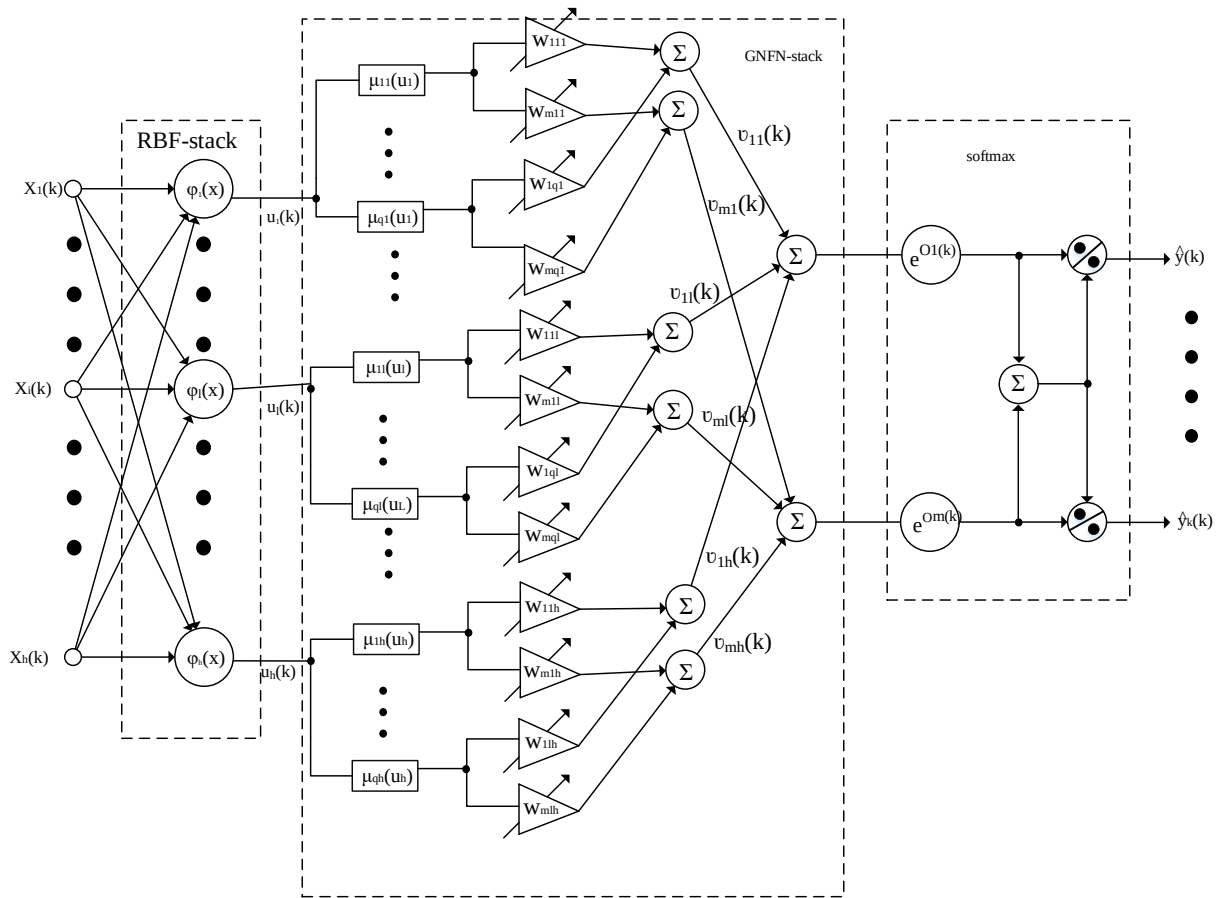


Fig. 1. Architecture of stacking hybrid system of computational intelligence based on kernel activation-membership functions

The GNFN in SHSCI contains qh kernel one-dimensional membership functions $\mu_{pl}(u_l)$, $p = 1, 2, \dots, q$; $l = 1, 2, \dots, h$; m tuned synaptic weights w_{jpl} , $j = 1, 2, \dots, m$ and $mh + m$ elementary adders, at the outputs of which signals $o_1(k), \dots, o_j(k), \dots, o_m(k)$ are formed, where m – the number of classes in the learning set

These signals are fed to the softmax activation functions at the output of the system, which calculate the level of fuzzy membership of each classified observation to a specific class $j = 1, 2, \dots, m$ in the form

$$\hat{y}_j(k) = \frac{e^{o_j(k)}}{\sum_{j=1}^m e^{o_j(k)}}.$$

In principle, instead of GNFN, a system of m ordinary neo-fuzzy neurons could be used, but in this case the number of membership functions would increase by m times, which would make the system as a whole more cumbersome.

As membership functions in NFN and GNFN, triangular functions that satisfy the Ruspini unity partitioning conditions are usually used. If these conditions are not met, then it is necessary to perform a defuzzification operation [26], which complicates the calculation process. Using B -splines instead of triangular structures improves the approximation qualities of the system, but also complicates the numerical implementation. Therefore, it was proposed [27] to use parabolic membership functions based on Epanechnikov kernels [28] instead of triangles, which provides better results compared to piecewise linear approximation when using triangles. In this case, the defuzzification operation is not required, since this procedure is implemented by the softmax activation function at the output of the system.

Membership functions of SHSCI are shown in Fig. 2.

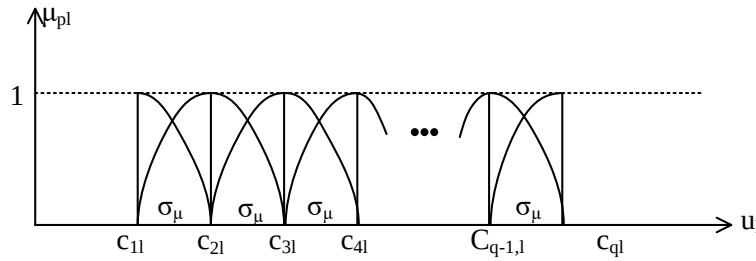


Fig. 2. Membership functions of SHSCI

The general form of the SHSCI membership functions is as follows:

$$\mu_{pl}(u_l) = \left[1 - \frac{(u_l - c_{pl})^2}{\sigma_\mu^2} \right]_+$$

where $[\cdot]_+ = \max\{0, \cdot\}$, which is also shown in Fig. 2.

The use of such membership functions leads to the fact that at each training cycle of the system not mqh synaptic weights are adjusted, but only $2mh$ weights, which naturally simplifies the training of the system as a whole.

Each of the membership functions $\mu_{pl}(u_l)$ is associated with m (the number of system outputs) synaptic weights w_{jpl} , which are to be adjusted during

the training process. These synaptic weights are associated with two layers of elementary summation blocks. The first layer contains mh adders and the second contains m adders. These adders calculate the values

$$v_{jl}(k) = \sum_{p=1}^q w_{jpl} \mu_{pl}(u_l(k)),$$

and

$$o_j(k) = \sum_{l=1}^h v_{jl}(k) = \sum_{l=1}^h \sum_{p=1}^q w_{jpl} \mu_{pl}(u_l(k)).$$

The signals from the output of the GNFN stack are fed to the SHSCI output layer, formed by the softmax activation function, resulting in the formation of a vector of outputs

$$\hat{y}(k) = (y_1(k), \dots, y_j(k), \dots, y_m(k))^T = \left(\frac{e^{o_1(k)}}{\sum_{j=1}^m e^{o_j(k)}}, \dots, \frac{e^{o_j(k)}}{\sum_{j=1}^m e^{o_j(k)}}, \dots, \frac{e^{o_m(k)}}{\sum_{j=1}^m e^{o_j(k)}} \right)^T,$$

each component of which specifies the level of fuzzy membership of each observation $x(k)$ to each of the classes $j = 1, 2, \dots, m$.

SHSCI training

Training of the proposed system can be conditionally divided into three relatively independent tasks: tuning of synaptic weights w_{jpl} , $j = 1, 2, \dots, m$; $p = 1, 2, \dots, q$; $l = 1, 2, \dots, h$ based on the concept of controlled learning with teacher; tuning of centers of kernel activation functions c_l , $l = 1, 2, \dots, h$; based on self-learning; and tuning of centers of membership

functions c_{pl} , $p = 1, 2, \dots, q$; $l = 1, 2, \dots, h$ also based on self-learning without a teacher.

The main task here is to tune the synaptic weights, which is related to optimizing the cross-entropy learning criterion.

$$E = - \sum_{k=1}^N \sum_{j=1}^m y_j^*(k) \ln y_j(k) = - \sum_{k=1}^N \sum_{j=1}^m y_j^*(k) \ln \frac{e^{o_j(k)}}{\sum_{j=1}^m e^{o_j(k)}},$$

where the components of the external reference signal that can take only two values: "1" or "0".

$$y_j^*(k), j = 1, 2, \dots, m$$

Considering the $(q_h \times 1)$ vector of membership function values

$$\mu(k) = (\mu_{11}(u_1(k)), \dots, \mu_{1l}(u_l(k)), \mu_{12}(u_2(k)), \dots, \mu_{pl}(u_l(k)), \dots, \mu_{qh}(u_h(k)))^T$$

and the vector of synaptic weights associated with the j -th output of SHSCI, we can write the signal value at the j -th output at the k -th tuning cycle in the form

$$w_j = (w_{j11}, \dots, w_{jq1}, w_{j12}, \dots, w_{jpl}, \dots, w_{jqh})^T,$$

$$o_j(k) = w_j^T (k-1) \mu(k),$$

$$\hat{y}_j(k) = \frac{e^{o_j(k)}}{\sum_{j=1}^m e^{o_j(k)}} = \frac{e^{w_j^T (k-1) \mu(k)}}{\sum_{j=1}^m e^{w_j^T (k-1) \mu(k)}}.$$

Optimization of the cross-entropy learning criterion using a gradient procedure leads to an algorithm for tuning synaptic weights of the form:

$$w_j(k) = w_j(k-1) + \eta_j(k)(y_j^*(k) - y_j(k))\mu(k),$$

where η is the learning rate parameter $\eta_j(k)$.

$$w_j(k) = w_j(k-1) + \frac{y_j^*(k) - y_j(k)}{\|\mu(k)\|^2} \mu(k) = w_j(k-1) + (y_j^*(k) - y_j(k))\mu^{T+}(k)$$

where $(\cdot)^+$ is the pseudoinversion symbol.

This algorithm can be given additional filtering (following) properties if it is modified in the following way [27]:

$$\begin{cases} w_j(k) = w_j(k-1) + \beta^{-1}(k)(y_j^*(k) - y_j(k))\mu(k), \\ \beta(k) = \alpha\beta(k-1) + \|\mu(k)\|^2, \end{cases}$$

where $0 \leq \alpha \leq 1$ is the forgetting (smoothing) factor. It is easy to see that $\alpha=0$ provides optimal speed, and $\alpha=1$ turns the algorithm into a stochastic approximation procedure.

The installation and adjustment of the centers of the kernel activation functions of the RBFN stack can be ensured by the combined use of the principle of "Neurons of data points" [31–32] and self-learning according to T. Kohonen [33–34]. The center of the first activation function c_1 is installed at a point with coordinates that coincide with the components of the first observation of the training sample $x(1)$, i.e. $c_1 = x(1)$.

Next, a certain threshold of indistinguishability of two neighboring centers is introduced and after the observation $x(2)$ arrives (it does not matter whether it is a vector or a matrix), the condition is checked. If this condition is true, then the first center is adjusted so that $\|x(2) - c_1\| < r$.

$$c_1(2) = \frac{c_1 + x(2)}{2}.$$

If the inequality is not satisfied, then this center is adjusted according to the self-learning rule "Winner Takes More": $r < \|x(2) - c_1(1)\| \leq 2r$

$$c_1(2) = c_1(1) + \eta_c(2)\psi_1(2)(x(2) - c_1(1))$$

with neighborhood function

$$\psi_1(x) = e^{-\|x(2) - c_1(1)\|^2 / 2r}.$$

In turn, this algorithm can be optimized for the rate of convergence by appropriate choice of the step. So, if we choose $\eta_j(k)$

$$\eta_j(k) = \|\mu(k)\|^{-2},$$

we arrive at the optimal adaptive algorithm in terms of speed by S. Kaczmarz, B. Widrow, M. Hoff [29, 30]:

If the inequality $\|x(2) - c_1(1)\| > 2r$ is not satisfied, then a second RBF activation function is formed

$$\varphi_2(x) = e^{-\|x - c_2\|^2 / 2\delta^2} = e^{-\|x - x(2)\|^2 / 2\delta^2}.$$

The following iterations regarding the formation of activation functions proceed in a similar way. Let us suppose that the system has already received k observations and formed $l < h$ activation functions, the last of which has the center $c_l(k)$. Let us suppose also that an observation of the image $x(k+1)$ has been received. Then, among all the already formed R -neurons, there is a winning neuron whose distance $\|x(k+1) - c_l^*(k)\|$ is minimal among all l formed R -neurons.

The correction of the centers cl is subsequently carried out as follows:

$$\text{If } \|x(k+1) - c_l^*(k)\| < r,$$

$$\text{then } c_l(k+1) = \frac{c_l^*(k) + x(k+1)}{2},$$

$$\text{if } r < \|x(k+1) - c_l^*(k)\| \leq 2r,$$

then

$$\begin{cases} c_l(k+1) = c_l^*(k) + \eta_c(k+1)\psi_l(k+1)(x(k+1) - c_l^*(k)) \\ \psi_l(x) = e^{-\|x - c_l^*(k)\|^2 / 2r} \end{cases}$$

if $\|x(k+1) - c_l^*(k)\| > 2r$, the next radial basis function is formed (a new R -neuron is added with center $c_{l+1}(k+1) = x(k+1)$).

This process continues until the maximum possible number h of kernel activation functions is formed in the first stack. It is easy to see that this approach is based on the ideas of evolving systems [13–15] and self-learning according to T. Kohonen [32].

As for the membership function $\mu_{pl}(u_l)$ in the CNFN stack, the adjustment of their centers c_{pl} is completely similar to the adjustment of the centers of the radial basis functions c_l . The only difference

is the choice of the indistinguishable thresholds $p\mu$. Fig. 3 shows the appearance of the membership functions after k -cycles of their adjustment.

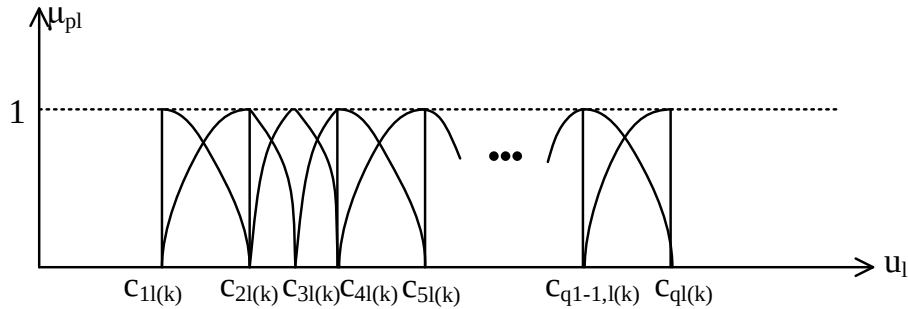


Fig. 3. Membership functions of CHSCI after k -cycles of their tuning

It is easy to see that these functions become asymmetric, and in analytical form they take the form

$$\begin{cases} \mu_{plL}(u_l) = \left[1 - \frac{(u_l - c_{pl}(k))^2}{(c_{pl-1}(k) - c_{pl}(k))^2} \right]_+, \\ \mu_{plR}(u_l) = \left[1 - \frac{(u_l - c_{pl}(k))^2}{(c_{pl+1}(k) - c_{pl}(k))^2} \right]_+, \end{cases}$$

where the indices L , R mean "left" and "right" relative to the center $c_{pl}(k)$.

It should also be noted that this method of self-learning membership function centers is computationally much simpler than setting up these same centers based on learning with a teacher [27], and these advantages are manifested in multidimensional systems, when the number of these membership functions can be quite large.

Results of the computational experiment

The objective of the computational experiment is to evaluate the performance of the proposed stacking hybrid system of computational intelligence in a pattern recognition task with online learning, focusing on classification accuracy, learning speed, and adaptability to data streams using short training samples.

The system was tested on benchmark datasets commonly used in pattern recognition:

- Fashion (Modified National Institute of Standards and Technology database): 28×28 grayscale test images of Zalando products (clothing, footwear, accessories that are presented on the Zalando platform), with 60000 training and 10000 test samples.

- CIFAR-10: 32×32 color images in 10 classes, with 50000 training and 10000 test samples.

Data samples were streamed to the system one-by-one or in small batches of 10–50 samples simulating real-time input.

The experimental settings are as follows:

1. Input format: matrix input was used without preliminary convolution, enabling direct use of image matrices in the RBF kernel layer.
2. System configuration:
 - maximum number of R -neurons in the first stack is $h = 100$;
 - number of membership functions per input channel is $q = 5$;
 - learning rate is $\eta = 0.01$;
 - forgetting factor is $\alpha = 0.1$;
 - indistinguishability thresholds for RBF and membership functions are $p_c = 0.1$, $p_\mu = 0.15$;
3. Evaluation metrics:
 - classification accuracy (overall and per class);
 - number of epochs or time to convergence;
 - evolution of the number of R -neurons and membership functions during online learning;
 - memory capacity (total parameters adjusted during learning).

The procedure of the computational experiment is as follows.

1. Initialization:
 - first R -neuron center c_1 was initialized using the first training sample;
 - membership function centers were initialized uniformly over the input range.

2. Online training:

- samples were fed to SHSCI sequentially;
- RBF centers and membership function centers were updated using T. Kohonen-style self-learning;
- synaptic weights w_{jpl} were updated using the S. Kaczmarz, B. Widrow, M. Hoff algorithm with pseudo-inverse and forgetting factor for filtering.

3. Testing:

- After each group of 500 samples, accuracy was evaluated on a fixed test set.
- The evolution of the network structure (number of R -neurons, active membership functions) was recorded.

The results of the computational experiment using Fashion MNIST and CIFAR-10 data samples are given in Table 1.

Table 1. The results of the computational experiment

Metric	Fashion MNIST	CIFAR-10
Accuracy (Final)	96.8%	84.3%
Accuracy after first 1000 samples	91.5%	74.0%
Number of R -neurons formed	87	93
Total membership functions active	340	365
Time to convergence	~4.8s	~6.1s
Memory capacity (parameters)	~16K	~18K

Additional results:

1. Training dynamics: the system quickly adapted to data, with most R -neurons and membership functions formed in the first 20% of data. Accuracy steadily improved with each batch.

2. Comparison with CNN:

- comparable accuracy to shallow CNNs but with faster training and significantly lower memory usage.
- better adaptation to streaming data and superior performance with small sample sizes.

Visualization of the computational experiment is presented below:

- evolution of membership functions (Fig. 4).
- online formation of R -neuron centers visualized with PCA (Principal Component Analysis) projection of input data space (Fig. 5).
- accuracy vs. number of processed samples plotted to compare learning speed with standard CNN (Fig. 6).

This figure illustrates how the membership functions in the SHSCI system become asymmetric over multiple online learning cycles. Initially symmetric, their shapes adapt based on data distribution, improving approximation accuracy without requiring defuzzification.

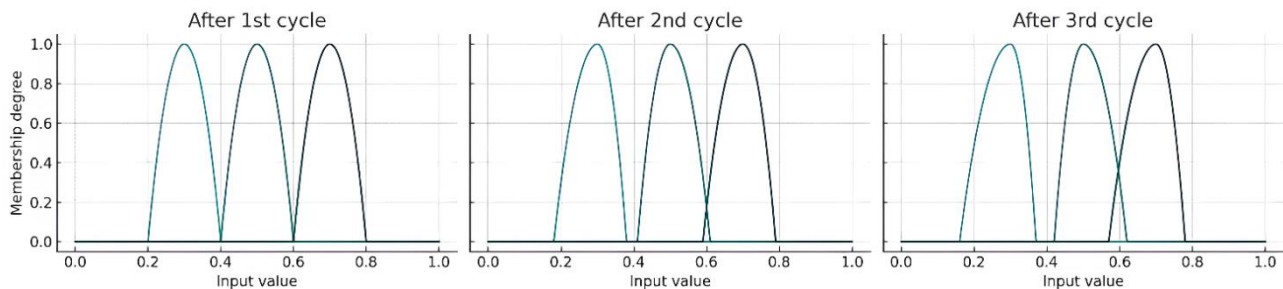


Fig. 4. Evolution of membership functions after k -cycles of tuning

This figure shows a PCA projection of the input data from a digit recognition dataset, for example, Fashion MNIST, with red cruciform markers representing the R -neuron centers dynamically formed during the online learning process of SHSCI. These centers act as kernels for the radial basis functions in the first stack, helping to separate overlapping class regions in the higher-dimensional space.

This plot shows that SHSCI achieves faster convergence in accuracy with fewer training samples compared to a standard CNN. This highlights SHSCI's efficiency in online learning scenarios and with limited data, making it well-suited for real-time tasks and stream-based pattern recognition tasks.

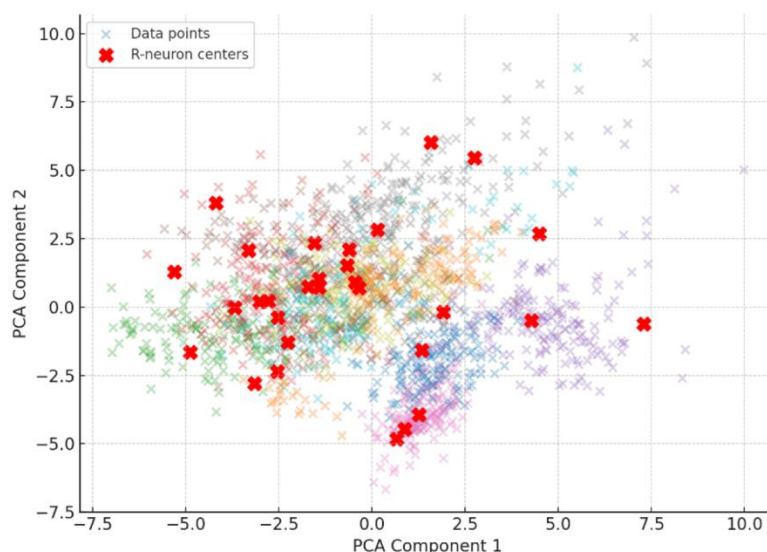


Fig. 5. PCA projection of data and R -neuron centers

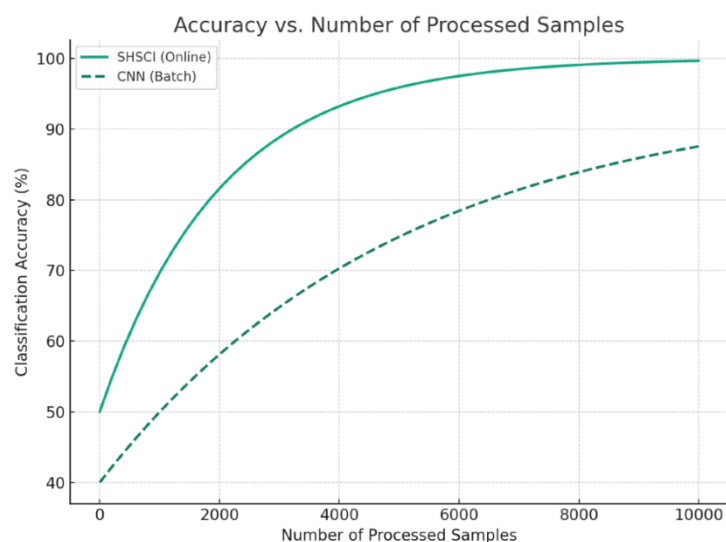


Fig. 6. Accuracy vs. number of processed samples plotted to compare learning speed with standard CNN

Conclusion

A stacking hybrid system of computational intelligence, the stacks of which are formed by a modified radial-basis-function neural network, which provides an increase in the dimensionality of the input space of signals-images and a generalized neo-fuzzy-neuron, which provides high approximating properties when restoring separating hypersurfaces between classes is proposed. The SHSCI is configured based on combined learning and self-learning, at the same time, not only synaptic weights are configured, but also the centers of kernel activation-membership functions. The process of configuring all parameters is quite simple in numerical implementation and occurs in the online mode as information is received for processing. The proposed

SHSCI is intended for solving problems of classification and recognition of images (pictures), at the same time, these images can be presented to the system input in both traditional vector and matrix forms. The advantages of the proposed system are manifested under conditions of short training samples, when traditional deep convolutional networks become ineffective, or when solving data stream mining problems, when training must be performed in real time.

In future work, the proposed stacking hybrid computational intelligence system can also be applied to the important practical problem of dynamic resource management in data networks as a pattern recognition task. In this context, pattern recognition is the transformation of an input signal (image/feature vector) into a class/label or output vector [35–36].

Conflict of interest

The authors declare that they have no conflict of interest, in particular financial, personal, authorial, or any other nature, that could influence the research, as well as the results published in this article.

Horizon 2020 MSCA RISE programme under grant agreement No. 10100828.

Data availability

The manuscript has no linked data.

Funding

This paper is part of the DIOR project that has received funding from the European Union's

Use of artificial intelligence

The authors confirm that they did not use artificial intelligence technologies to write this work.

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Received (Надійшла) 14.02.2026

Accepted for publication (Прийнята до друку) 21.05.2026

Publication date (Дата публікації) 27.06.2026

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СТЕКОВА ГІБРИДНА СИСТЕМА ОБЧИСЛЮВАЛЬНОГО ІНТЕЛЕКТУ НА ОСНОВІ ЯДЕРНОЇ АКТИВАЦІЙНОЇ ФУНКЦІЇ ТА ЇЇ ОНЛАЙН-НАВЧАННЯ В ЗАДАЧІ РОЗПІЗНАВАННЯ ОБРАЗІВ

Предмет дослідження. Предметом дослідження є стекова гібридна система обчислювального інтелекту на основі ядерної активаційної функції, що поєднує переваги нейронних мереж і нечітких моделей для розв'язання задач розпізнавання образів в умовах невизначеності, зашумленості та обмеженої розмірності наборів даних. **Мета дослідження.** Мета роботи полягає у розробці гібридної стекової системи з ядерною функцією активації та онлайн-навчанням для забезпечення кращої точності, стабільності й швидкості розпізнавання шаблонів у нестационарних середовищах у режимі реального часу. **Завдання.** Розробити архітектуру стекової гібридної системи; запропонувати ядерну активаційну функцію та алгоритм її параметризації; побудувати алгоритм онлайн-навчання з покроковим оновленням параметрів; дослідити властивості

збіжності та обчислювальної складності; провести експериментальне порівняння з базовими моделями машинного навчання та класичними нейро-нечіткими підходами. **Методи.** Застосовано методи машинного навчання, ансамблеві та стекові підходи, ядерні методи, адаптивну оптимізацію в онлайн-режимі, статистичний аналіз якості класифікації, моделювання та обчислювальні експерименти на синтетичних і реальних наборах даних для задач розпізнавання образів. **Результати.** Запропонована модель забезпечує вищу точність класифікації та кращу узагальнювальну здатність порівняно з окремими базовими моделями, демонструє стійкість до шумів і змін розподілів даних, а також швидку адаптацію в умовах потокових даних завдяки онлайн-навчанню. Показано зменшення похибки та стабільну роботу системи в нестационарних середовищах. **Висновки.** Розроблена стекова гібридна система з ядерною активаційною функцією є ефективним інструментом для задач розпізнавання образів у режимі реального часу. Поєднання стекового підходу та онлайн-навчання підвищує точність, стійкість і адаптивність системи, що робить запропонований підхід перспективним для практичного застосування в інтелектуальних інформаційних системах і кіберфізичних середовищах.

Ключові слова: інтелектуальний аналіз потокових даних; машинне навчання; нейро-нео-фаззі система; обчислювальний інтелект; стекова гібридна система; ядерна активаційна функція.

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Золотухін О. В., Бодянський Є. В., Філатов В. О., Кудрявцева М. С., Василюк О. О. Стекова гібридна система обчислювального інтелекту на основі ядерної активаційної функції та її онлайн-навчання в задачі розпізнавання образів. *Сучасний стан наукових досліджень та технологій в промисловості*. 2026. № 2 (36). С. 40–51. DOI: <https://doi.org/10.30837/2522-9818.2026.2.040>

Zolotukhin, O., Bodyanskiy, Y., Filatov, V., Kudryavtseva, M., Vasylets, O. (2026), "Stacking Hybrid System of Computational Intelligence Based on Kernel Activation-Membership Functions and its Online Learning in Pattern Recognition Task", *Innovative Technologies and Scientific Solutions for Industries*, No. 2 (36), P. 40–51. DOI: <https://doi.org/10.30837/2522-9818.2026.2.040>